



Evaluation of Nonlinear Dynamic Response of Rigid and Semi-Rigid Steel Frames Under Far-Field Earthquake Records

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Abstract

The purpose of this research was to evaluate the nonlinear dynamic response of rigid and semi-rigid steel frames under Far-Fault Earthquake Records. Accordingly, the fragility curve of the moment frames with rigid and semi-rigid connections was determined. Considering the analytical knowledge of structures in the past, the analysis and design of steel frames based on the assumptions of rigid or joint connections. While laboratory studies show that most connections are semi-rigid and due to the importance of connections in the structures, it is very important to recognize and accurately study their behavior, especially during an earthquake, and their design must be under their real structural behavior. For this purpose, three two-dimensional steel moment frame structures with 6, 12, and 18 stories were used, which represent short, medium, and high structures. Considering the rigid and semi-rigid connections, their seismic performance was investigated using the nonlinear dynamic incremental analysis (IDA). Three cases of connections have been selected corresponding to 50, 60, and 70% rigidity. Finally, the collapse fragility curve parameters obtained and compared. According to the obtained results, decreasing the rigidity of the beam-to-column connections increases the dispersion of the collapse fragility curve. Besides, it was observed that considering the semi-rigid connections leads to a reduction of the median of the collapse fragility curve. The result shows that the mentioned difference cannot be neglected.

Keywords: Steel Moment Frame, Rigid and Semi-Rigid Connections, Collapse Fragility Curve, Incremental Dynamic Analysis, Far-Field earthquake Records.

1. Introduction

The study of the global collapse was triggered by considering P- Δ effects on seismic response. Currently, the collapse fragility curve is the most important and accepted tool for evaluating the collapse of the structure. A set of IDA analyses can play a vital role in determining the estimation parameters and in turn determine the collapse fragility curve. Incremental Dynamic Analysis (IDA) was invented to take the inherent variability of earthquakes into account during the seismic response analysis of structures [1-5].

In recent years, the effect of the severe earthquake occurrence in designing steel structures especially moderate-to-high rise buildings, directed the most designers towards the design of the structures with rigid moment connections, since such structures had a ductility and significant earthquake resistance [6].

After the 1994 Northridge earthquake and 1995 Kobe earthquake, a significant number of buildings using a steel momentum frame system with welded rigid connections were destroyed from the beam-to-column connection and many structures, in contrast to what was expected, lost their overall performance. One of the main causes of these failures is low ductility and stress concentration in the welded area of connections [7]. Subsequently, many studies have been carried out on the formation of fusion connections in the high-hazard regions. One of the suggestions in this regard is the use of semi-rigid connections. This connection in severe earthquakes on the one hand, is effective in the depletion of earthquake energy through proper rotational ductility and on the other hand, this connection is effective in reducing earthquake loads by increasing its slope [8].

In the designing of the multi-story buildings, gravity loads (dead loads, live loads) are not so problematic,

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but lateral loads due to wind or earthquakes are very important. Lateral loads can create critical stress and unpleasant seismic structures, as well as large lateral displacements, which concern the residents. In many earthquake countries, steel frames with semi-rigid connections or reinforced concrete frames with shear walls are carried out to reduce the adverse effects of lateral forces. In this research, three steel moment frames were used to analyze, modeling and investigating the nonlinear dynamic behavior of structures under different earthquake records. Seismo-Struct software has been used for nonlinear dynamics analysis. The results of the nonlinear analysis, the collapse fragility curve of the structures are plotted and compared using story drift.

In the last two decades, the attention of the Society of Structural and Earthquake Engineers has attracted more to the actual behavior of the structure and the nature of earthquake characteristics due to the devastating earthquakes in the world. Considering the high seismic hazard of the region of Iran as well as the population growth in recent years and the need for construction in the high-hazard region, the construction and earthquake engineers sought to study the structures in the near and far fault as a necessary issue. Considering that the occurrence of the damages in the most steel structures due to the earthquakes is in connections, designing and implementation of connections are very important as the main elements of the building. Any weaknesses in them can have irreparable consequences. Therefore the issue of the semi-rigid connection is raised. Since joints are one of the most important and vital parts of the moment frame structures, their behavior during the earthquake must be identified.

Regarding the similar studies in this field, examined 16 laboratory samples for measuring upper and lower shield using web angle through the cyclic test. They investigated the effects of several parameters in the connection, such as the diameter of the screws, the beam length and the other parameters that can be affected on the anchorage and rotation-stiffness. can be mentioned as a practical and applied research to use in the design of structures with web angle and se at angle semi-rigid connections [9]. Researchers discussed shield positioning leech action under cyclic loading and changes in geometric and mechanical characteristics of the connection, as well

as the pre-stress effects of the screws and the coefficient of friction between the connecting components on moment-rotation curve in detail in the form of six tests. This study showed that the lever force depends on the diameter of the screws, the distance between the screw and the thickness of the wings, and the shield time. This study also showed that the pre-stressing of the screws increases the stiffness of the connections. Swanson et al. carried out two full-scale experiments on upper and lower shield connections along with web angle. The first connections were subjected to a combination of shearing and bending to obtain the rotation-moment curve. The results were compared with the six experimental samples performed by Lyon and Evanson [10-12]. The main purpose of this research was to evaluate the connections, especially the semi-rigid connections and comparing these connections with rigid connections on the better behavior of these connections in terms of collapse fragility curve. According to the definition, the collapse of a building during or shortly after earthquake stimulation, as a result of the loss of structural integrity of the building, is due to the demand for force and a large displacement change in one or more components of the structural system of the building. The excessive seismic demand for structural strength reduces the strength and stiffness of structural elements, which can lead to the general or local collapse of the building. The overall design of a structure may have different causes. The expansion of the initial failure from one element to another, may lead to its collapse [13-14].

2.Research Methodology

2.1. The Considered Structural Models

Three steel frames with rigid and semi-rigid connections subjected to the 7 Far-Field ground motions records were considered and designed according to the ASCE 7-10 code requirements [13]. The selection of the building is based on the different period range which is included mid-to-high rise buildings. These structural models are assumed to be of administrative buildings type with the same plan dimensions, located in a high seismic site at Tehran with site class D according to the ASCE 7-10 [13].

The seismic parameter A was considered 0.35, respectively; the importance factor (I) of 1, the response modification factor (R) of 5 (6- and 12-story) and 7.5 (18-story) were considered. The structural system of the 6- and 12-story frames (S-6 and S-12) is intermediate moment resisting frames, while the 18-story frame (S-18) is considered as special moment resisting frame. The beam and column sections were selected I-shape and box-shape, respectively. The yielding strength of steel is

2400 kg/cm² in all buildings. In all models, the ultimate strength of longitudinal bars is 3700 kg/cm². ETABS (2013) was used to design the structural models [15].

The studied buildings have a 36 m×18 m rectangle plan with a story height of 4.1 m and spans of 6m. In designing the building models the story drift ratios were limited to values specified by the considered code. Figure 1 shows typical plan of buildings and the selected frame were considered in this study.

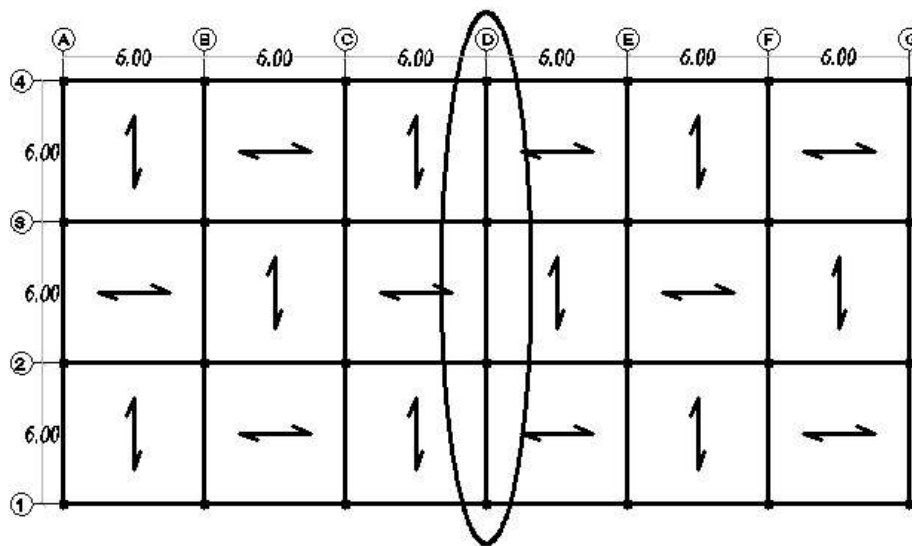


Fig.1. Frame plan of the studied building

In this research, the fragility curves of the studied 2-D frames were determined for the following four cases: 50-, 60-, 70- and 100% rigidity in the beam-column connections. To make the semi-rigid connections with different percentages (50-, 60- and 70% rigidity), the

width of the upper and lower beam flanges were reduced in the length of L in the two ends of the all beams. Tables 1-3 show the properties of structural elements and length of the parameter of L.

Table 1. Sections of rigid and semi-rigid 6-story frame structures

NO	STORY	COLUMN	BEAM	RIGID b_f	S – R %70 b_f	S – R %60 b_f	S – R %50 b_f	L cm
1	S-1	B50*50*20	PG350B200T20	200	140	120	100	90
2	S-2	B45*45*20	PG350B200T20	200	140	120	100	90
3	S-3	B45*45*20	PG350B200T20	200	140	120	100	90
4	S-4	B40*40*18	PG350B200T20	200	140	120	100	90
5	S-5	B40*40*18	PG330B180T15	180	125	110	90	90

6	S-6	B35*35*15	PG300B180T10	180	125	110	90	90
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Table 2

Sections of rigid and semi-rigid 12-story frame structures

NO	STORY	COLUMN	BEAM	RIGID b_f	S – R %70 b_f	S – R %60 b_f	S – R %50 b_f	L cm
1	S-1	B55*55*20	PG370B200T20	200	140	120	100	100
2	S-2	B55*55*20	PG370B200T20	200	140	120	100	100
3	S-3	B50*50*20	G370B200T20	200	140	120	100	100
4	S-4	B50*50*20	G370B200T20	200	140	120	100	100
5	S-5	B45*45*18	G350B200T15	200	140	120	100	100
6	S-6	B45*45*18	G350B200T15	200	140	120	100	100
7	S-7	B40*40*18	G350B200T15	200	140	120	100	100
8	S-8	B40*40*18	G330B200T15	200	140	120	100	100
9	S-9	B35*35*15	G330B200T15	200	140	120	100	100
10	S-10	B35*35*15	G330B200T15	200	140	120	100	100
11	S-11	B30*30*15	G270B180T10	180	125	110	90	100
12	S-12	B30*30*15	G270B180T10	180	125	110	90	100

Table 3

Sections of rigid and semi-rigid 18-story frame structures

NO		COLUMN	BEAM	RIGID b_f	S – R %70 b_f	S – R %60 b_f	S – R %50 b_f	L cm
1	S-1	B60*60*30	PG400B300T20		210		150	
2	S-2	B60*60*30	PG400B300T20	300	210	180	150	120
3	S-3	B60*60*30	PG400B300T20	300	210	180	150	120
4		B55*55*30		300		180		120
5		B55*55*30		300		180		120
6	S-6	B55*55*30	PG370B300T20	300	210	180	150	120
7		B50*50*30		300		180		120
8	S-8	B50*50*30	PG370B300T20	300	210	180	150	120
9	S-9	B50*50*30	PG350B300T20		210		150	
10	S-10	B45*45*25	PG350B300T20	300	210	180	150	120
11	S-11	B45*45*25	PG350B300T20	300	210	180	150	120
12	S-12	B45*45*25	PG330B300T20	300	210	180	150	120
13	S-13	B40*40*25	PG330B300T20	300	210	180	150	120
14	S-14	B40*40*25	PG330B300T20	300	210	180	150	120
15	S-15	B40*40*25	PG330B300T20	300	210	180	150	120
16	S-16	B35*35*20	PG300B300T20	300	210	180	150	120
17	S-17	B35*35*20	PG300B300T20	300	210	180	150	120
18	S-18	B35*35*20	PG300B300T20	300	210	180	150	120

ETABS (2013) software has been used for static linear designing and modeling of structures. Then, Seismo-Struct 2016 software has been used for modeling and analyzing nonlinear incremental dynamic (IDA) to investigate the dynamic behavior of structures under earthquake mapping. The basis of this software is based on nonlinear analysis using fiber elements. As known, adjusting and using

plastic joints for nonlinear analysis of structures is a hard and time-consuming work. The advantage of using fiber elements is to predict the location and range of plastic joints by the software itself so that it is no longer necessary to adjust the curve of the shift behavior of the plastic joints for any structural analysis and to be given to the software.

2.2.Numerical Modeling

Following the selection and normalization of the earthquake records and preparation of the structural models, the incremental dynamic analysis (IDA) was conducted using Seismo-Struct 2016 software under the applied all seismic excitations. The IDA curves were developed considering the scalar intensity measures (IM), i.e., $S_a(T_1, 5\%)$.

This software is based on nonlinear analysis using the fiber section, and considers the spread of plasticity along the element. This is the most economical and accurate approach to investigate the seismic behavior of structures. The advantage of using fiber sections is to predict the location and range of plastic joints by the software itself so that it

is no longer necessary to adjust the curve of the shift behavior of the plastic joints for any structural analysis and to be given to the software.

Accordingly, in this study, the member was divided into several smaller members for the analysis. The bending and axial behavior of the section is obtained by using these small elements and integrating them throughout the member. In the nonlinear fiber model, the section is divided into several small elements. The number of fibers used in each section is 150. Figure 2 presents the steel sections divided into the number of elements. Bilinear model is used for modeling of the steel in Seismostruct. The strain-hardening ratio is considered equal to 0.005. [16]

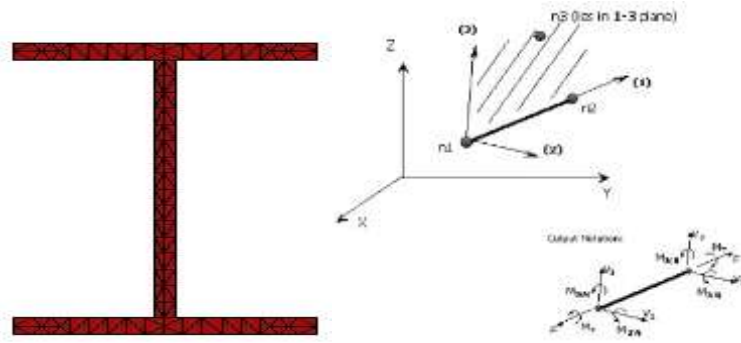


Fig 2. Geometric properties of the element [16]

The Riley method with 5% damping and the number of modes that have a total of 95% mass participation in the response of the structures have been used to consider the viscous damping in the buildings. In

this method, the viscous damping region is considered in the linear and nonlinear as a combination of structural mass and stiffness. Equation (1) expresses this combination.

$$C = \alpha M + \beta K \quad (1)$$

In which, C, M, and K matrices are damping, mass, and stiffness matrices. Changes in the

damping ratio with frequency are shown in Figure (3) [17].

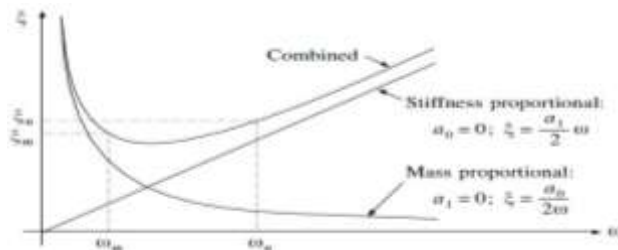


Fig 3. Relation between damping ratio and frequency for Rayleigh damping [17]

In Table 4 presents the results of the period of the studied frames. These results are based on nonlinear

models made in the Seismo-Struct software.

Table 4
Fundamental period of the studied frames

Structures Studied	T %100		T %60	
S-6	1.158	1.249	1.292	1.348
S-12	2.439	2.661	2.767	2.898
S-18	2.912	3.063	3.132	3.216

2.3. Analysis of connections with reduced cross section using incremental dynamic analysis

Researchers described the dynamic analysis method more in detail and compared the structures of the earthquake intensity response to the structure response (a 20-story steel frame, a five-story curtain steel frame and a three-story moment frame with fragile connections). They also suggested ways to efficiently implement an increasing dynamic analysis and summarize information from different curves obtained for various earthquakes. They observed that increasing dynamic analysis is a valuable tool for simultaneously obtaining seismic demands on structures and their overall collapse capacity [1]. Incremental dynamic analysis is a parametric method in which one or more seismic records, each one, are scaled to a specific intensity and applied to the structure. In addition to

investigating the seismic behavior of the structure, this method also shows the structural capacity and can also be used to determine the seismic performance of the structures. The purpose of the IDA is to plot the Damage Measure (DM) values at each level (each stage of the analysis) against the Intensity Measure of Scalable Earthquakes (IM). The IDA curve is depicted in terms of Damage Size (DM) against one or more Intensities (IMs) based on two or more independent IMs [18]. As mentioned, Seismo-Struct software was used to analyze the effect of the semi-rigid connections in collapse fragility curves using a stranded model (finite element). Initially, the structures with rigid connections were analyzed using IDA. Then, the same structures were again analyzed with the semi-rigid connections by reducing the width of upper and lower beam flanges in three states, 50-, 60- and 70 percent of rigidity in connections.

3. Analysis and Results

3.1. IDA curve

It is concluded that earthquakes with a high frequency and intensity were a major role in the financial distress caused by earthquakes in earthquake damage estimates. On the other hand, high-intensity earthquakes and low occurrence of earthquakes are important in terms of casualties. Therefore, an analytical method should be used to determine its response across all functional limits to investigate the performance of the structure. For this purpose, an incremental dynamic analysis (IDA), in which the intensity of the earthquake in the fundamental period of the structure with damping of 5% from zero to the extent that it leads to its collapse, is applied to the structure [19]. The first step in assessing the collapse of a building is to obtain an IDA curve from the IDA analysis results. The selection of intensity measures (IM), like EDP, depends on the efficiency in terms

of seismic intensities. The IDA curves were developed considering the $Sa(T_1, \zeta=5\%)$ as scalar intensity measures (IM). To plot an IDA curve for a moment frame resistance under an arbitrary earthquake record, at each seismic intensity level ($IM = im_i$), the maximum internal drift ratio (IDR) in all stories and all steps of the IDA should be obtained. By repeating this process for other IMs, a set of points is obtained as $(im_i, \max IDR_i)$, which produces the IDA curve by plotting the points [1]. Since in this research, the performance of structures is considered until the collapse level, the IM range continues to the maximum possible value for the structure to reach the dynamic state of instability or collapse. Figure 4 shows the IDA curve for a 6-story structure with rigid connections. As can be seen, there are 14 IDA curves for 7 pairs of earthquake

records. Therefore, the collapse fragility curve is obtained based on these curves.

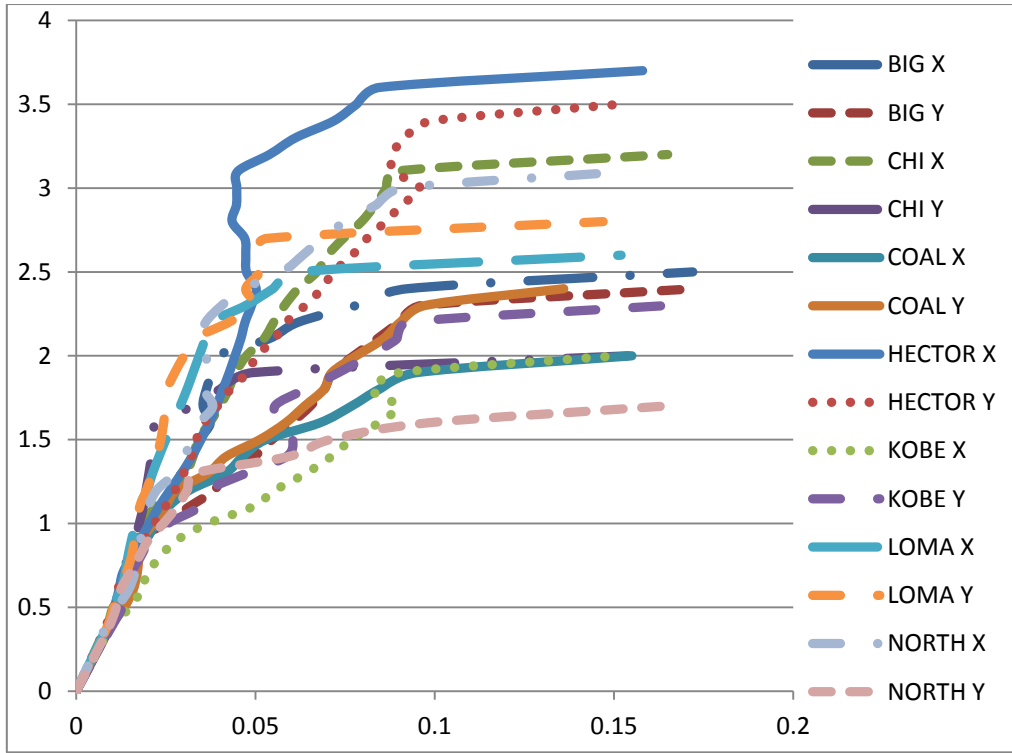


Fig 4. IDA curve of 6-story of 2-D frame with the rigid connections

3.2. Estimation Of Collapse Fragility Curves Of The Studied Structures

To extract the occurrence probability of collapse from IDA results, the so-called fragility curves are used. Collapse fragility curve can be considered as a cumulative distribution function (CDF) of a stochastic variable namely collapse capacity (S_{ac}). Ibarra and Krawinkler showed that S_{ac} points follow a log-normal distribution i.e. $\ln(S_{ac}) \rightarrow N(\eta_C, \beta_{RC})$ where η_C and β_{RC} are median collapse capacity and dispersion of collapse capacity values due to

different earthquake records which are numerically equal to the standard deviation of collapse capacity values [20]. For a given hazard level, like PR, corresponding spectral acceleration can be obtained using seismic hazard curves and collapse probability can be calculated from Equation (2), where η_C and β_{RC} are median and standard deviation of log-normal cumulative distribution function, respectively:

$$P(C|S_a^{PR}) = \Phi\left(\frac{\ln(S_a^{PR}) - \ln(\eta_C)}{\beta_{RC}}\right) \quad (2)$$

Figures 5, 6 and 7, Comparison of mean and standard deviation of frame present the collapse

fragility curves of the studied buildings under different rigidity percent in connections.

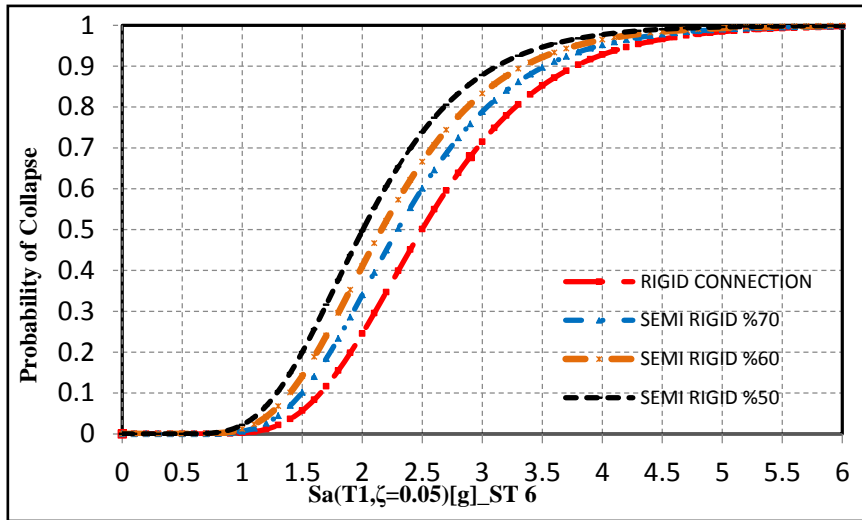


Fig 5.Collaps fragility curves of 6-story frame considering $IM=Sa (T_1, \xi = 0.05)$

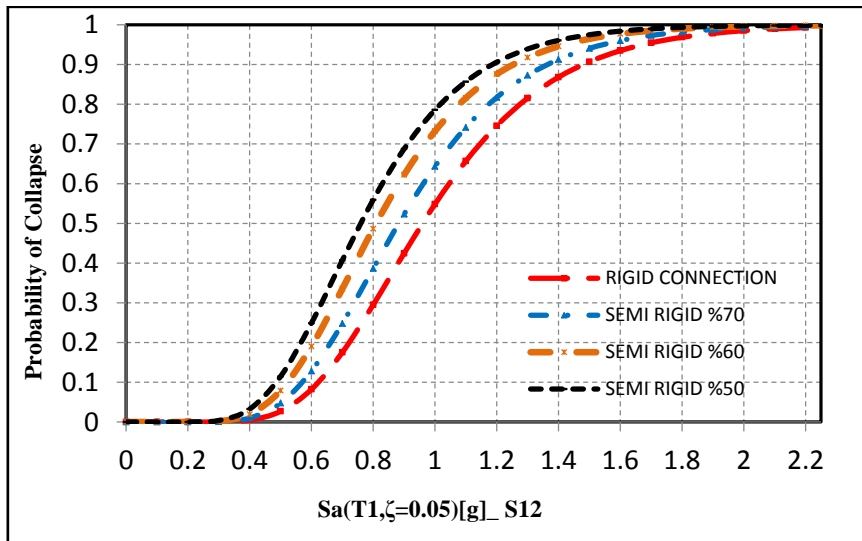


Fig 6. Collapse fragility curves of 12-story frame considering $IM=Sa (T_1, \xi = 0.05)$

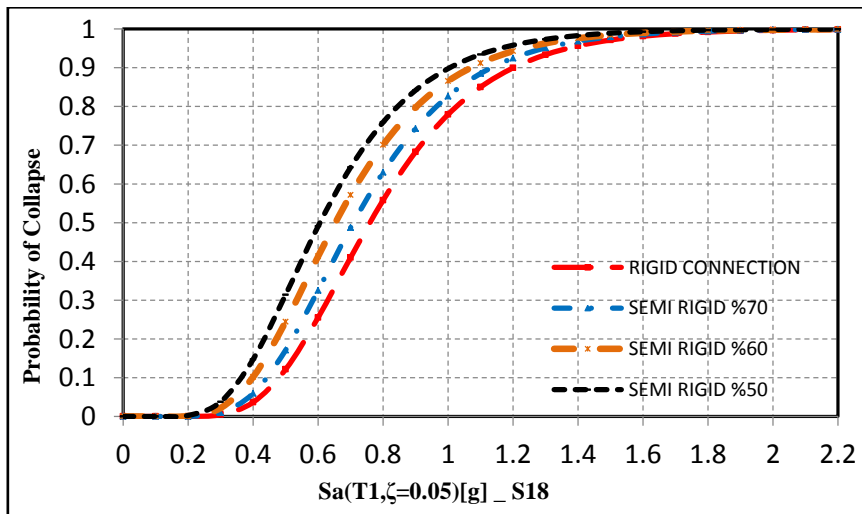


Fig 7.collaps Fragility curves of 18-story frame with $IM=Sa (T_1, \xi = 0.05)$

Since the fragility curves are in the form of log-normal cumulative distribution function with median (η_C) and standard deviation (β_{RC}) parameters, the

fragility curve parameters are summarized in Table 5.

Table 5
The collapse fragility curve parameters of studied frames in different scenarios

Number of story	Rigidity percentage		Standard deviation β_{RC}
6-story	%100	2.42	0.322
	%70	2.18	0.334
	%60	2.06	0.340
	%50	1.826	0.345
12-story	%100	0.925	0.338
	%70	0.872	0.340
	%60	0.802	0.341
	%50	0.730	0.348
18-story	%100	0.760	0.357
	%70	0.735	0.366
	%60	0.676	0.385
	%50	0.628	0.395

Figures 5-7 and Table 5 present that the median collapse capacity of the fragility curve decreases and dispersion of fragility curve increases by decreasing rigidity percentage. Decreasing in rigidity percentage of connections up to 50% in 6-story structure, make

decreasing in the median of the collapse capacity in the range of 10-24.5%. Also, Decreasing in rigidity percentage of connections up to 50% in 12- and 18-story structure, make decreasing in the median of the collapse capacity in the range of 3.5-21.08%.

3.3.Comparison of Mean and Standard Deviations Values of Structures With Equal Number of Variables and Rigidity

Based on the results obtained from Figures (8) to (11), the fragility curve of 6-, 12- and 18-story frames with the same rigidity percentage is observed in which the median of the collapse capacity is decreases by increasing the number of story.

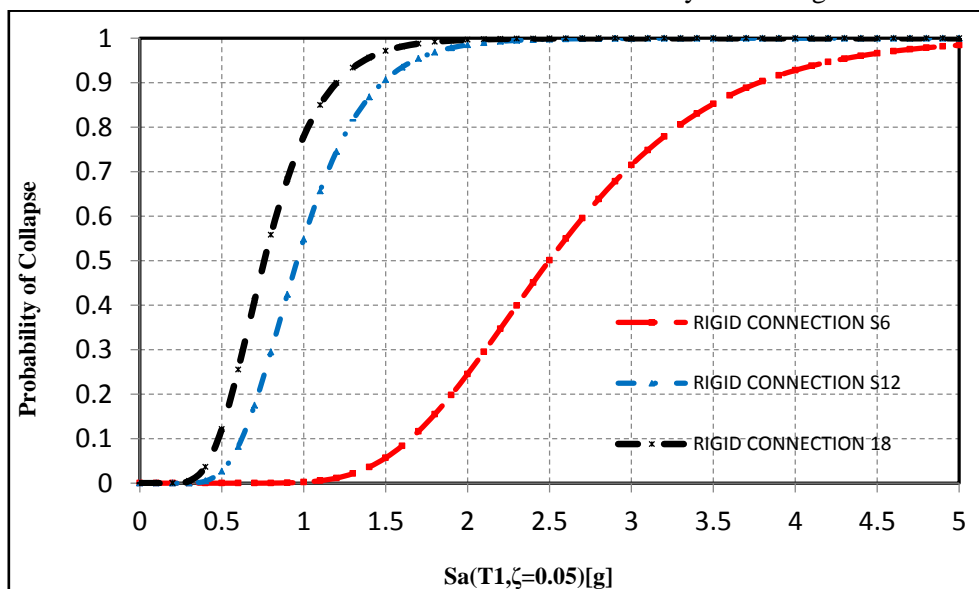


Fig 8. Collapse fragility curves of 6-, 12- and 18-stories frame considering $IM=Sa(T_1, \xi = 0.05)$ and rigid connections

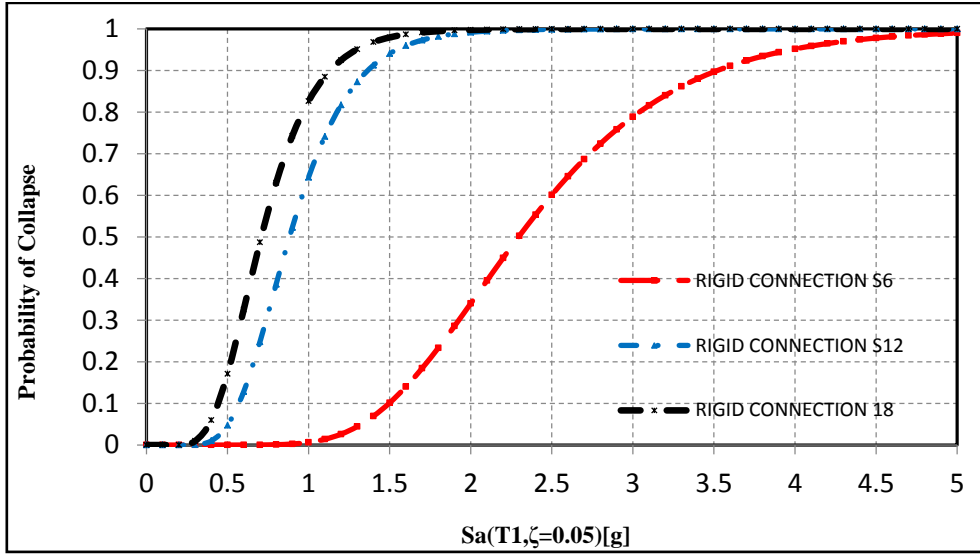


Fig 9. Collapse fragility curves of 6-, 12- and 18-stories frame considering $IM=Sa(T_1, \zeta = 0.05)$ and semi-rigid connections (70% rigidity percentage)

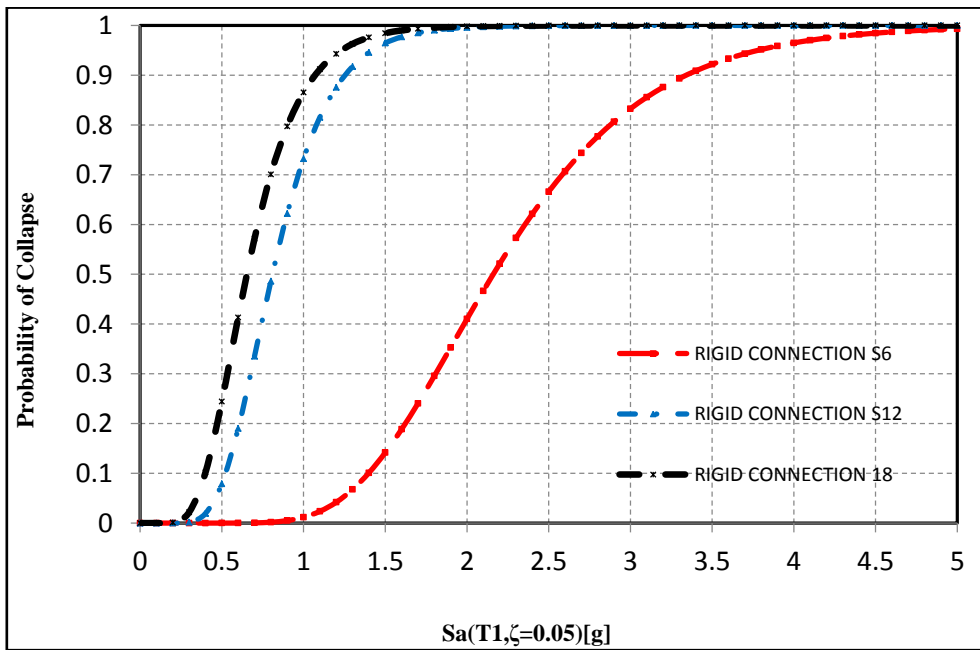


Fig 10. Collapse fragility curves of 6-, 12- and 18-stories frame considering $IM=Sa(T_1, \zeta = 0.05)$ and semi-rigid connections (60% rigidity percentage)

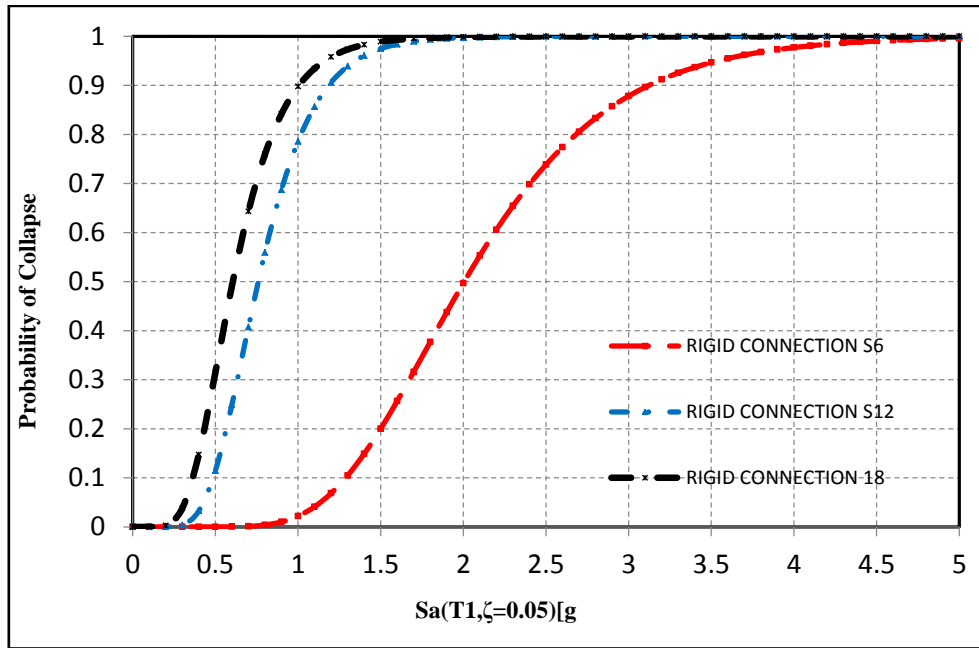


Fig 11. Collapse fragility curves of 6-, 12- and 18-stories frame considering $IM=Sa (T_1, \xi = 0.05)$ and semi-rigid connections (50% rigidity percentage)

4. Conclusion

Assuming non-linear behavior for steel materials, this study modeled three 6-, 12- and 18- story steel moment resisting frames. Incremental dynamic analysis (IDA) was conducted to take the uncertainties of percentage rigidity and earthquake records into account. The buildings performance was studied for rigid and semi-rigid connections using seismic demand probabilistic analysis. In addition, the effect of the different rigidity percentage of connections in collapse fragility curve was evaluate. It could be concluded that the reduction in percentage rigidity of connections shifts the collapse fragility curve to the left and reduces the median of collapse fragility curve. It should be noted that no fixed period range is selected and it can be different from structure to structure. Additionally, the

fundamental period of frames with semi-rigid connections increases compared with rigid frames. Also, collapse in all studied structures with different stories and different rigidity percentage, occurs on the lowest story in building.

Additionally, in 6-story building (short structure), rigid-connections assumption leads to an error range by 10% – 24.5% in median collapse fragility curve which is negligible compared to semi-rigid connections. It should be noted that error range in mid-to-high rise buildings was decreased compare with short buildings. This difference cannot be neglected. Moreover, decreasing the rigidity percentage in connections up to 50% of rigidity, decreases the median of collapse capacities and increases the seismic vulnerability of the building.

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